

# Outline

- ▶ Relational Algebra (6.1)
- ▶ E/R Model (7.2 - 7.4)
- ▶ E/R Diagrams (7.5)
- ▶ Reduction to Schema (7.6)
- ▶ Relational Database Design (7.7)
- ▶ Functional Dependencies (8.1 – 8.4)
- ▶ Normalization (8.5 – 8.7)

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## Functional Dependencies

- ▶ Difference between holding on an *instance* and holding on *all legal relations*

Title	Year	Length	inColor	StudioName	prodC#	StarName
Star wars	1977	121	Yes	Fox	128	Hamill
Star wars	1977	121	Yes	Fox	128	Fisher
Star wars	1977	121	Yes	Fox	128	H. Ford
King Kong	1933	100	no	RKO	20	Fay

- ▶ **Title** → **Year** holds on this instance
- ▶ Is this a true functional dependency ? **No.**
  - Two movies in different years can have the same name.
- ▶ Can't draw conclusions based on a *single instance*
  - Need **domain knowledge to decide which FDs hold**

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# FDs and Redundancy

- ▶ Consider a table:  $R(\underline{A}, B, C)$ :
  - With FDs:  $B \rightarrow C$ , and  $A \rightarrow BC$
  - So “A” is a Key, but “B” is not
- ▶ So: there is a FD whose left hand side is not a key
  - **Leads to redundancy**

Since B is not unique, it may be duplicated  
Every time B is duplicated, so is C

Not a problem with  $A \rightarrow BC$   
A can never be duplicated

A	B	C
a1	b1	c1
a2	b1	c1
a3	b1	c1
a4	b2	c2
a5	b2	c2
a6	b3	c3
a7	b4	c1

Not a duplication  $\rightarrow$  Two different tuples just happen to have the same value for C

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# Functional Dependencies

- ▶ Functional dependencies and *keys*:
  - A *key constraint* is a specific form of a FD.
  - E.g. if  $\alpha$  is a superkey for  $R$ , then:
    - ▶  $\alpha \rightarrow R$
    - Similarly for *candidate keys* and *primary keys*.
- ▶ Deriving FDs
  - A set of FDs may imply other FDs
  - e.g. If  $A \rightarrow B$ , and  $B \rightarrow C$ , then clearly  $A \rightarrow C$
  - We will see a formal method for inferring this later

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# Definitions

- ▶ 1. A relation *instance*  $r$  *satisfies* a set of functional dependencies,  $F$ , if the FDs *hold* on that relation
- ▶ 2.  $F$  *holds on* a relation *schema*  $R$  if no legal (allowable) relation instance of  $R$  violates it
- ▶ 3. A functional dependency,  $\alpha \rightarrow \beta$ , is called *trivial* if:
  - $\alpha$  is a superset of  $\beta$
  - e.g. **MovieName, length  $\rightarrow$  length**
- ▶ 4. Given a set of functional dependencies,  $F$ , its *closure*,  $F^+$ , is all the FDs that are implied by FDs in  $F$ .

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# Approach

- ▶ 1. We will encode and list all our knowledge about the schema
  - Functional dependencies (FDs)
  - Also:
    - Multi-valued dependencies (briefly discuss later)
    - Join dependencies etc...
- ▶ 2. We will define a set of rules that the schema must follow to be considered good
  - “Normal forms”: 1NF, 2NF, BCNF, 3NF, 4NF, ...
  - A normal form specifies constraints on the schemas and FDs
- ▶ 3. If not in a “normal form”, we modify the schema

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# BCNF: Boyce-Codd Normal Form

- A relation schema  $R$  is “in BCNF” if:
  - Every functional dependency  $\alpha \rightarrow \beta$  that holds on it is *EITHER*:
    - 1. Trivial *OR*
    - 2.  $\alpha$  is a superkey of  $R$
- Why is BCNF good ?
  - Guarantees *no redundancy* because of a functional dependency
  - Consider a relation  $r(A, B, C, D)$  with functional dependency  $A \rightarrow B$  and two tuples:  $(a1, b1, c1, d1)$ , and  $(a1, b1, c2, d2)$
  - $b1$  is repeated because of the functional dependency
  - BUT this relation is not in BCNF  
 $A \rightarrow B$  is neither trivial nor is  $A$  a superkey for the relation

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## BCNF and Redundancy

- ▶ Why does redundancy arise ?
  - Given a FD,  $\alpha \rightarrow \beta$ , if  $\alpha$  is repeated ( $\beta - \alpha$ ) has to be repeated
  - 1. If rule 1 is satisfied, ( $\beta - \alpha$ ) is empty, so not a problem.
  - 2. If rule 2 is satisfied, then  $\alpha$  can't be repeated, so this doesn't happen either
- ▶ Hence no redundancy because of FDs in BCNF
  - Redundancy may exist because of other types of dependencies
    - Higher normal forms used for that (specifically, 4NF)
  - Data may naturally have duplicated/redundant data
    - We can't control that unless a FD or some other dependency is defined

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# Approach

- ▶ 1. We will encode and list all our knowledge about the schema:
  - Functional dependencies (FDs); Multi-valued dependencies; Join dependencies etc...
- ▶ 2. We will define rules the schema must follow to be “good”
  - “Normal forms”: 1NF, 2NF, 3NF, BCNF, 4NF, ...
  - A normal form specifies constraints on the schemas and FDs
- ▶ 3. If not in a “normal form”, we modify the schema
  - Through lossless decomposition (splitting)
  - Or direct construction using the dependencies information

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## BCNF

$r(A, B, C, D)$  with  $A \rightarrow B$  and:  
(a1, b1, c1, d1), and (a1, b1, c2, d2)

- ▶ What if the schema is not in BCNF ?
  - *Decompose (split) the schema into two pieces.*
- ▶ From the previous example: split the schema into:
  - $r_1(A, B)$ ,  $r_2(A, C, D)$
  - The first schema is in BCNF, the second one may not be (and may require further decomposition)
  - No repetition now:  $r_1$  contains (a1, b1), but b1 will not be repeated
- ▶ Careful: you want the decomposition to be **lossless**
  - *No information should be lost*
    - The above decomposition is lossless
  - We will define this more formally later

$R_1$	$R_2$
(a1, b1)	(a1, c1, d1)
	(a1, c2, d2)

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# Normalization

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## BCNF

- ▶ Recall that  $R$  is in BCNF if every FD,  $\alpha \rightarrow \beta$ , is either:
  - 1. Trivial, or
  - 2.  $\alpha$  is a *superkey* of  $R$
- ▶ *No redundancy*
  
- ▶ What if the schema is not in BCNF ?
  - *Decompose (split) the schema into two pieces.*
  - Careful: you want the decomposition to be lossless

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# Achieving BCNF Schemas

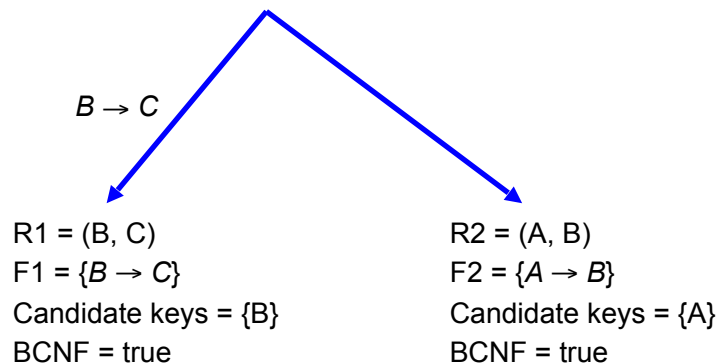
- ▶ For all dependencies  $\alpha \rightarrow \beta$  in  $F^+$ , check if  $\alpha$  is a superkey
  - (attribute closure)
- ▶ If not, then
  - Choose a dependency in  $F^+$  that breaks the BCNF rules, say  $\alpha \rightarrow \beta$
  - Create  $R_1 = \alpha\beta$
  - Create  $R_2 = R - (\beta - \alpha)$ .
  - Note that:  $R_1 \cap R_2 = \alpha$  and  $\alpha \rightarrow \alpha\beta$ , so:
    - $\alpha$  is a *superkey* of  $R_1$
    - *lossless decomposition* (lossless if intersection of two attribute sets is key for one)
- ▶ Repeat for  $R_1$  and  $R_2$ 
  - Define  $F_i$  to be all dependencies in  $F^+$  that contain only attributes in  $R_i$

**Note:**  
 $(R - (\beta - \alpha)) \equiv (R - \beta)$   
 if no extraneous attributions in FDs  
*We use  $(R - \beta)$  in this course.*

# Achieving BCNF Schemas

Example 1

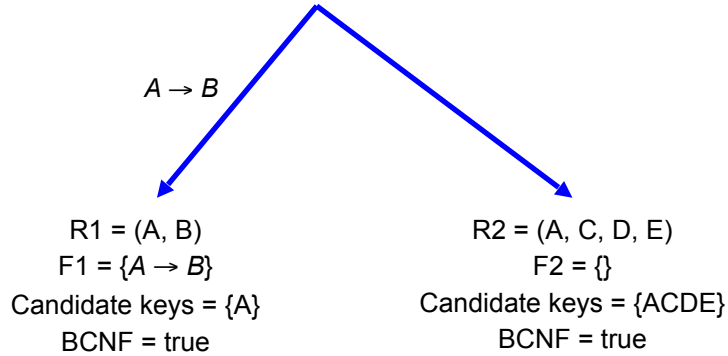
$R = (A, B, C)$   
 $F = \{A \rightarrow B, B \rightarrow C\}$   
 Candidate keys =  $\{A\}$   
 BCNF? No.  $B \rightarrow C$  violates.



- Dependency-preserving?
  - yes

Example 2a

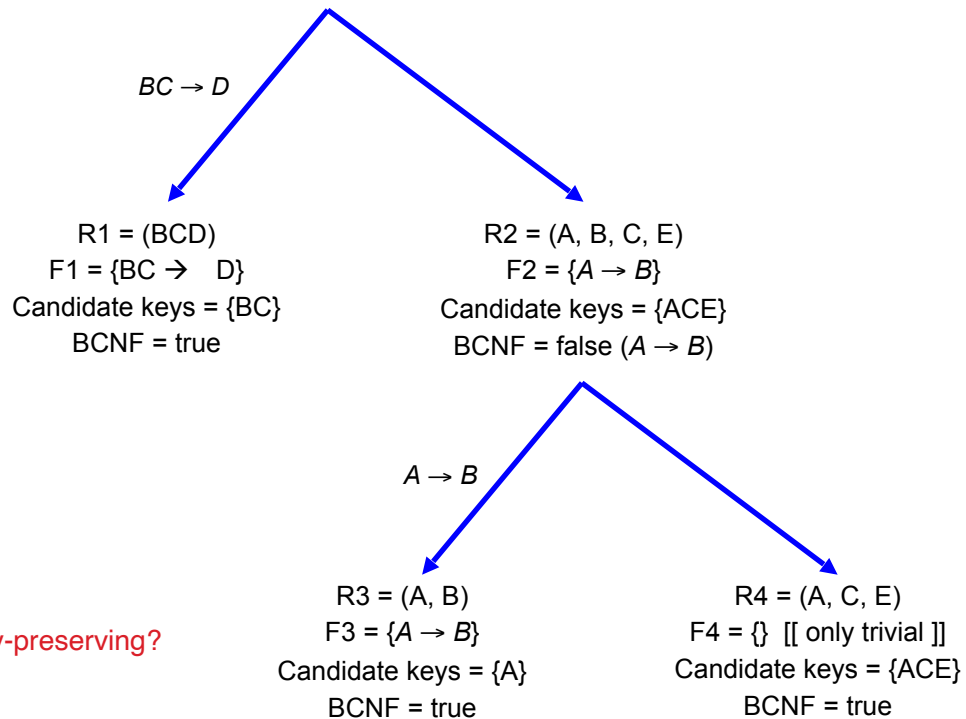
$R = (A, B, C, D, E)$   
 $F = \{A \rightarrow B, BC \rightarrow D\}$   
 Candidate keys =  $\{ACE\}$   
 BCNF = Violated by  $\{A \rightarrow B, BC \rightarrow D\}$



- Dependency-preserving?
  - no: lost  $BC \rightarrow D$

Example 2b

$R = (A, B, C, D, E)$   
 $F = \{A \rightarrow B, BC \rightarrow D\}$   
 Candidate keys =  $\{ACE\}$   
 BCNF = Violated by  $\{A \rightarrow B, BC \rightarrow D\}$

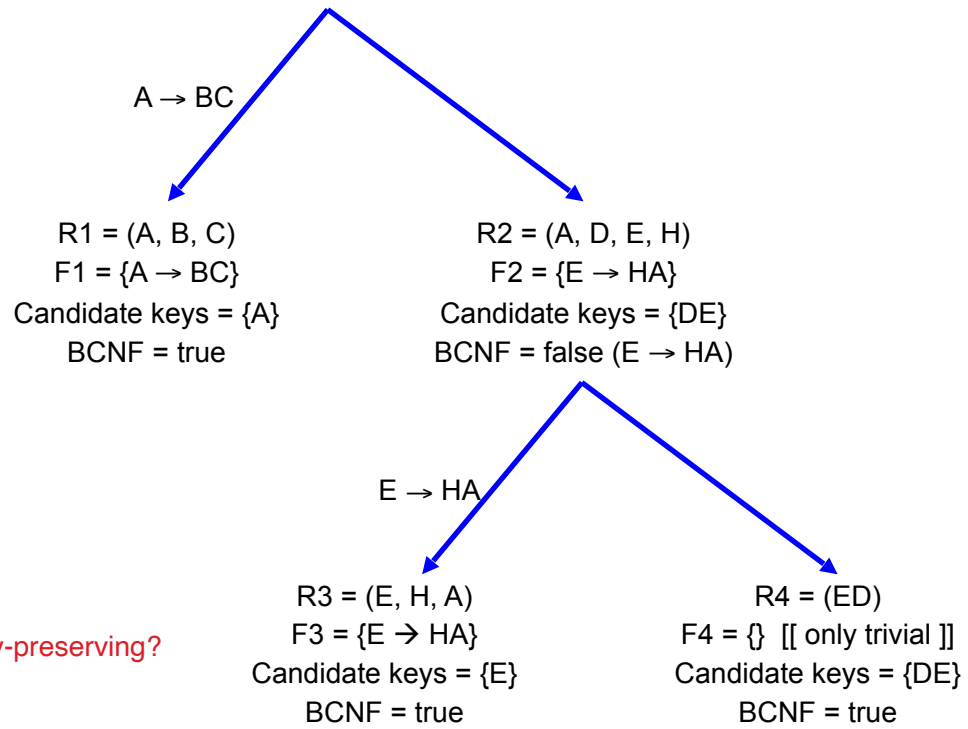


- Dependency-preserving?
  - yes



Example 3

$R = (A, B, C, D, E, H)$   
 $F = \{A \rightarrow BC, E \rightarrow HA\}$   
Candidate keys =  $\{DE\}$   
BCNF = Violated by  $\{A \rightarrow BC\}$  and  $\{E \rightarrow HA\}$



- Dependency-preserving?
- yes